THE ROYAL SOCIETY

The motion of geophysical vortices

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The thin fluid layers constituting the Earth's oceans and atmosphere give rise to a wide variety of long-lived vortices, which are important in determining, amongst other things, our weather and climate. On a rotating planet, these vortices have the ability to self-propagate. This paper investigates this phenomenon by describing the fundamental physics involved and reviewing our current understanding of it. Also, areas of future research related to the motion of geophysical vortices are speculated upon.

Keywords: vortices; oceanography; climate; eddies; meteorology; fluid dynamics

1. Introduction

In the first year of publication of this journal, Robert Hooke (Hooke 1665) reported on his astronomical observations of a 'Prominency of the Belt' in the atmosphere of Jupiter (see also Robert Boyle's article in *Phil. Trans. R. Soc. Lond.*, no. 14 (1666), for further early discussion and speculation regarding the origin and nature of the anomalies in Jupiter's atmosphere). We now realize this anomaly is a vast atmospheric vortex, known as the Great Red Spot, which continues to swirl coherently and vigorously today, more than 300 years after publication of Hooke's observations. Unlike a fluid with the freedom to flow in three-dimensions in an arbitrarily chaotic way, such as a plume of smoke rising from a lit cigarette, the Jovian atmosphere is confined to a thin† shell in which the vertical velocities are negligible compared with the horizontal velocities. A remarkable feature of such quasi-two-dimensional flows is the emergence of localized, swirling, laminar flows (vortices) from a sea of turbulence (see, for example, McWilliams 1990; Polvani *et al.* 1994). The Earth's ocean and atmosphere are also thin fluid layers and are, therefore, dynamically similar to Jupiter's atmosphere, and, similarly, spawn coherent vortices. For example, the atmosphere contains the wintertime stratospheric polar vortex, a particularly robust structure in which a region of stratospheric air is effectively isolated from the rest of the atmosphere, enabling the chemistry of ozone depletion to take place (McIntyre 1995). Tropical cyclones (also known as hurricanes or typhoons) and tornadoes are examples of intense atmospheric vortices with sufficiently rapid swirling flows to cause devastation in their path. Indeed, Hurricane Mitch claimed over 20 000 lives in parts of central America in October 1998.

The ocean also has an equally diverse range of vortices. In the Gulf Stream region off the east coast of North America, coherent 'blobs' of both anomalous warm and

† The actual depth of Jupiter's atmosphere is not well known. However, its dynamics appear to be well modelled by various 'thin-layer' (i.e. shallow water) based theories (see, for example, Williams 1984).

cold water—Gulf Stream rings—are observed. Figure 1 shows a satellite image of the sea surface temperature in this region in which both cold and warm Gulf Stream rings are visible. These rings form when the meandering of the Gulf Stream becomes sufficiently intense to break off and form separate eddies. Similar vortices are also observed in the Kuroshio current region off Japan and in the Agulhas current region at the tip of South Africa. Gulf Stream rings are examples of vortices that are located near the surface of the ocean (hence their distinct surface signatures in figure 1), but ocean vortices may exist at any depth within the fluid column. For example, Mediterranean salt lenses (Meddies) are large flat 'discs' of anomalously warm and salty water of Mediterranean origin found in the North Atlantic ocean. Typically, they have a diameter of 50 km, a thickness of several hundred metres and lie about 1000 m below the surface of the ocean and have lifetimes of up to two years (Armi *et al*. 1989). In the abyssal ocean, there is also good evidence that the dense water formed in the polar oceans is spread by vortices rather than continuous currents (see, for example, Whitehead 1989; Lane-Serff & Baines 1998). On a smaller scale, the coastal surf zone gives rise to vortices with length-scales from 10 to 100 m, which may, for example, cause rip currents.

The ability of ocean vortices to transport heat, salt and momentum over large distances, together with their anomalous biological and chemical properties, make them important components of the global ocean circulation. For example, Agulhas eddies are responsible for the transport of at least 2.2×10^{20} J yr^{-1} of heat and at least 14×10^{12} J yr^{-1} of salt from the Indian to the Atlantic Ocean (Duncombe Rae *et al*. 1996), and, thus, they form an important link in the global 'conveyer belt' of the world's ocean circulation. Meddies also serve in the dispersal of significant amounts of relatively salty water from the Mediterranean throughout the north Atlantic. How such vortices redistribute heat and salt throughout the global ocean determines, to a large extent, our climate. Unfortunately, since typical diameters of ocean vortices range from tens to hundreds of kilometres, they are difficult, if not impossible, to resolve with present-day climate models. Thus, understanding their behaviour is vital so that their effects can be successfully parametrized in such models. An example of the possible importance of vortices in biology is the suggestion (Copley 1998) that vortices formed by the instability of deep-sea hydrothermal plumes may be responsible for the trapping and dispersal of exotic biota throughout the abyssal ocean. At a much smaller scale, in coastal regions, small scale surf-zone vortices may be important in the dispersal of pollutants. In the atmosphere, the role of atmospheric vortices in determining our everyday weather, together with the destructive power of tropical cyclones and tornadoes, provide ample motivation to their study.

Not surprisingly, geophysical vortices have received considerable attention from oceanographers, meteorologists and climate scientists. They are also of considerable appeal to theoreticians working in fluid dynamics because of their laminar flow properties, their high degree of radial symmetry and their inherent nonlinearity (a necessary ingredient in order to prevent their dispersion).

A fundamental aspect of the study of geophysical vortices is the prediction of their trajectories. It is also a very challenging problem given the many different factors that may influence their propagation. For example, it seems certain that thermodynamics plays a significant role in the propagation of tropical cyclones. Gulf Stream rings are also influenced by thermodynamics along with, for example, the presence of neighbouring rings, the Gulf Stream itself, bottom topography and friction. An important effect on all but the smallest of geophysical vortices (such as surf-zone vortices and tornadoes; see discussion in § 2) is that of the rotation of the Earth. In addition to the restriction of vertical velocities imposed by the thinness of geophysical fluids, rotation imparts a rigidity, or elasticity, to the fluid in the poleward direction. As will be discussed in §§ 2 and 3 of this paper, this elasticity allows the existence of large-scale fluid oscillations (Rossby waves) and also, remarkably, provides a mechanism for geophysical vortices to self-propagate, or drift. They are not simply pushed about by the prevailing winds or currents. It is this phenomenon which this paper will concentrate on. For example, even in the absence of external currents, Meddies propagate at a speed of the order of several centimetres per second. Over a typical lifetime of two years, this amounts to a displacement of the order of 2000 km.

This paper reviews and discusses recent advances in our understanding of the propagation of geophysical vortices through the use of analytical and numerical models and laboratory experiments. In particular, it will concentrate on the mechanism for vortex drift on a rotating planet and related effects. It will also speculate on areas of future research. Throughout this article, the focus will be upon monopole vortices, i.e. those with a single set of closed streamlines in which the main circulation is in one sense only. Observations of the Earth's oceans and atmosphere show that monopoles are the most commonly occurring type of vortex, but it is important to note that there is observational evidence for dipoles (i.e. two counter-rotating vortices), of which atmospheric blocking events are one such example. Dipoles are important structures both theoretically and experimentally and reviews treating their properties can be found in Flierl (1987) and van Heijst (1994), the latter of which also discusses tripoles. It should also be noted that there are many other important aspects in the study of geophysical vortices that will not be addressed here, for example their genesis, stability, thermodynamics and the effects of friction.

2. Fundamentals: some effects of rotation

The rotation of the Earth, Jupiter and other planets has a significant effect on their vortices. Rotation gives rise to Coriolis forces and also imparts a rigidity, or elasticity, to the fluid in the poleward direction. These effects are discussed in this section.

(a) The Coriolis force

It is natural to describe the dynamics of the atmosphere and ocean from the point of an observer fixed relative to the surface of the Earth or the rotating planet under consideration. The equations of motion for a thin layer of fluid in such a frame of reference are based on Newton's second law with the inclusion of the Coriolis force owing to the choice of the non-inertial (i.e. rotating) frame of reference. For a particle moving tangentially to the planet's surface (that is, horizontally, or perpendicular to the local gravity vector) the magnitude of the Coriolis force, F_{Coriolis} is equal to twice the product of the momentum of the particle mv with the component of the planet's angular velocity vector in the direction of the local vertical. This component varies with the sine of the latitude ϕ so that it is at a maximum at the poles and vanishes at the equator (see figure 2). Thus,

$$F_{\text{Coriolis}} = fmv, \tag{2.1}$$

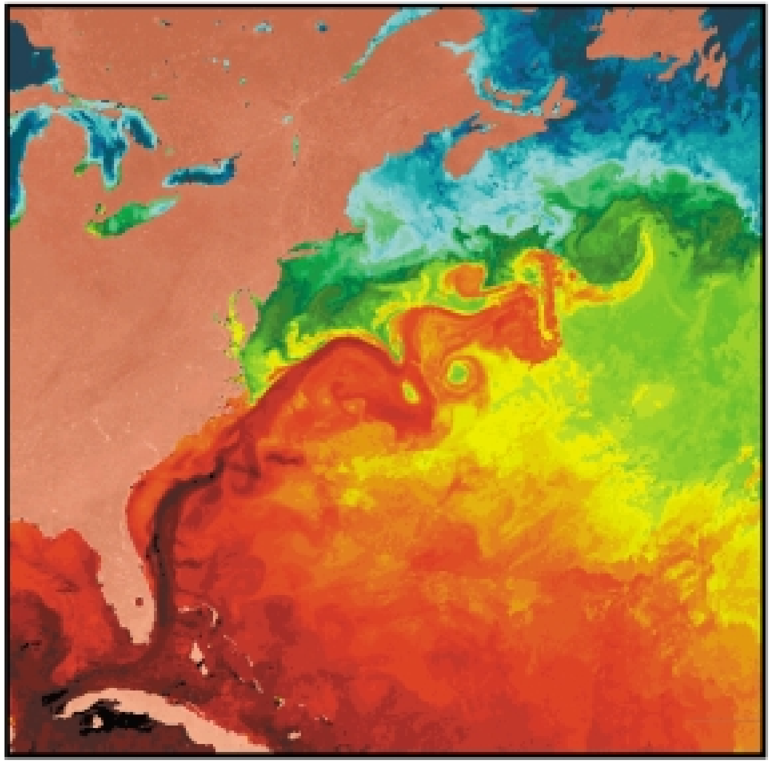

Figure 1. Satellite image of the Gulf Stream region. Red indicates warm water and blue cold water. The sharp boundary between the two indicates the Gulf Stream. Note the anomalous (green) patch of cold water to the south of the Gulf Stream and the warm (red) patch to its north. These are Gulf Stream rings.

where $f = 2\Omega \sin\phi$ is known as the Coriolis parameter and Ω is the angular velocity of the Earth. Importantly, the direction of the Coriolis force acting on the particle is also in the horizontal plane but at right angles to the direction of the velocity. In the Northern (Southern) Hemisphere, it acts to the right (left) looking in the direction of motion. (In vector cross-product notation, $\boldsymbol{F}_{\mathrm{Coriolis}} = -2mf\boldsymbol{k} \times \boldsymbol{v}$, where $\boldsymbol{v}$ is the velocity vector and $\boldsymbol{k}$ is the local vertical unit vector.)

(*b*) *The Rossby number*

The effect of rotation on a vortex can be gauged through the magnitude of the Rossby number *Ro*, which measures the relative importance of inertia of the swirling flow of the vortex to the Coriolis acceleration: $Ro = V/fL$. Here, V is a typical swirl velocity associated with the vortex, f is the Coriolis parameter, and L is the length-scale of the vortex. If *Ro* is of the order of unity or less, then rotational effects are significant in the vortex dynamics. For example, typical values for Agulhas rings at latitude 35°S are $f = 8 \times 10^{-5}$, $V = 50\,\mathrm{cm\,s^{-1}}$ and $L = 80\,\mathrm{km}$, giving $Ro \approx 0.1$, indicating that the Earth's rotation plays a significant role in its dynamics. On the other hand, tornadoes with their large velocities and relatively small size have large *Ro*, so that the Earth's rotation plays only a minor role in their dynamics. Likewise,

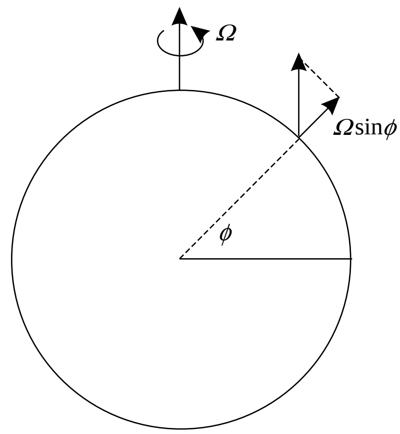

Figure 2. The variation with latitude of the local vertical component of the Earth's rotation vector. It is maximum at the poles and vanishes at the equator.

the familiar 'bath-tub' vortex has a Rossby number of at least 10^3, and, therefore, the Earth's rotation is insignificant in its dynamics. This destroys the myth that water will disappear down a sink in opposing senses according to which hemisphere the sink is in. In fact, the sense of swirl of the draining water will depend more strongly on factors such as the sense of any residual angular momentum the fluid has when the plug is pulled, and the shape of the bath.

To a large extent, the motion in both the atmosphere and ocean are small Rossby number flows and are in a state of *geostrophic* balance. That is, the Coriolis force balances the pressure gradient. Given that fluid tends to move from regions of high pressure to low pressure, as it does so it will experience a Coriolis force. In particular, in the Northern Hemisphere, fluid tends to spiral clockwise as it moves out from a high-pressure cell and spiral anticlockwise as it moves towards a region of low pressure. Therefore, in the Northern Hemisphere cyclones (low-pressure cells) have anticlockwise circulation and anticyclones (high-pressure cells) have clockwise circulation. The sense of these circulations is reversed in the Southern Hemisphere.

(*c*) *The conservation of potential vorticity*

At the heart of understanding the dynamics of geophysical fluids is the conservation of potential vorticity Q. This law is derived from the equations of motion for a thin layer of fluid on a rotating planet in the absence of friction. Its precise form depends on the details of the fluid model being considered: for a single layer of homogeneous fluid, the potential vorticity is

$$Q = (f + \omega)/h, \tag{2.2}$$

where h is the depth of the fluid layer, which can vary owing to, for example, topography, and ω is the relative vorticity. The relative vorticity measures the local circulation or 'spin' of the fluid about the vertical: positive relative vorticity implies local anticlockwise (clockwise) circulation in the Northern (Southern) Hemisphere and negative relative vorticity implies local clockwise (anticlockwise) circulation in the Northern (Southern) hemisphere. Indeed, local anomalies of Q and, hence, ω,

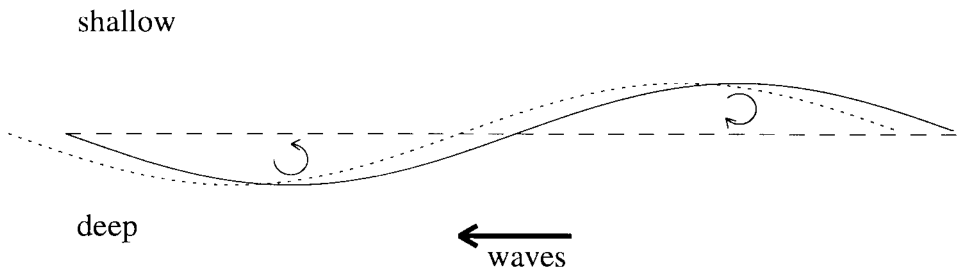

Figure 3. The Rossby wave mechanism. The dashed line along $y = 0$ separates shallow ($y > 0$) from deep ($y < 0$) water. The solid line represents an initial perturbation to the line of fluid particles initially overlaying the step. Relative vorticity production (see text), of the sign as indicated by the arrowed circles, advects the perturbation to the left.

induce localized circulations, and, therefore, geophysical vortices tend to be synonymous with signatures in the potential vorticity field. An important property of Q is that knowledge of its distribution everywhere enables the velocity field to be calculated, which in turn can be used to advect the potential vorticity distribution. Thus the potential vorticity field controls the dynamical evolution of the fluid.

As mentioned in § 1, the presence of rotation imparts a 'stiffness' in the fluid in the meridional direction (i.e. the direction of changing ϕ or north–south direction). More generally, as is evident in equation (2.2), this 'stiffness' is in the direction of the gradient of Q, which, for a quiescent fluid (i.e. $\omega = 0$) with constant depth ($h =$ const.), corresponds to the meridional direction (i.e. increasing ϕ) toward the North Pole since $Q = f/h$ (recall $f = 2\Omega \sin\phi$). To see this, consider the simple case where variations in Q due to latitude may be ignored compared with variations in topography, i.e. $Q \approx f_0/h$, where $f_0 = 2\Omega \sin\phi_0 > 0$ is the constant Coriolis parameter at some latitude ϕ_0 in the Northern Hemisphere. In particular, consider a fluid of constant shallow depth for $y > 0$ and constant, but deeper, depth for $y < 0$ (see figure 3). Thus the line $y = 0$ separates two regions of differing Q, with higher Q for $y > 0$ and lower Q for $y < 0$. The gradient of Q is, thus, positive in the positive y direction so that it mimics the variation of Q on the spherical Earth if increasing y is identified with increasing latitude ϕ.

If the interface is perturbed as shown in figure 3, the conservation of potential vorticity (equation (2.2)) implies that fluid that has moved from deep to shallow water will acquire negative relative vorticity ω, and, therefore, will start to circulate clockwise. On the other hand, fluid that has moved from shallow to deep water acquires positive relative vorticity ω and circulates anticlockwise. The effect of the circulations is to cause the disturbance to propagate to the left, as shown in figure 3. The conservation of potential vorticity implies the existence of such waves whenever there exists a gradient in the background Q field. As in the above description, this gradient may be provided by the presence of variable topography but, equally, may be due to the meridional variation in f (i.e. the sphericity of the planet), or may be provided by the presence of sheared currents, i.e. variations in ω such as may occur near the Gulf Stream for example. Such waves are called Rossby waves and are always such that in the Northern Hemisphere they propagate with higher values of Q on their right facing in the direction they are travelling.

The ubiquitous presence of the meridional gradient in f as well as sheared currents and variable topography mean that Rossby waves of one sort or another are ever present in the Earth's atmosphere and ocean. Such waves have important implications for the propagation of vortices, since any translating vortex must disturb the background Q field, which, in turn, leads to the radiation of Rossby waves. Analogous to a ship moving on the water's surface, such radiation will lead to a loss of energy, or drag, on the vortex. The consequences of this radiation, and models to study its effect, will be discussed in §§ 3 and 4.

(*d*) *The westward drift of a vortex*

In the absence of variable topography and ambient winds and currents, the meridional variation in the Coriolis force causes vortices to self-propagate in a westward direction; or, more precisely, in the opposite sense to the planet's rotation. Figure 4*a*, *b* shows a vortex consisting of an isolated lens of fluid of anomalous density embedded in a motionless fluid, i.e. there is no flow of fluid exterior to the vortex and we call the vortex isolated.† In the Northern Hemisphere, associated with such a vortex is a clockwise circulation, i.e. it is an anticyclone. As fluid swirls within the vortex it experiences a Coriolis force: as fluid moves eastward in the northern half of the vortex there is a net Coriolis force directed southward, and as fluid moves westward in the southern half of the vortex the Coriolis force is northward (see figure 4*a*). Since the magnitude of the Coriolis force increases in a northward direction, the Coriolis force experienced by the top half of the vortex is greater than that experienced by the bottom half of the vortex. This is indicated in figure 4*a* by the differing length of the north–south arrows. Thus, this motion alone leads to a net southward force acting on the vortex. If the vortex motion is steady, there can be no net force on the vortex and the vortex must, therefore, drift westward as a whole (figure 4*b*) in order to balance the force due to the swirling motion in figure 4*a*. Since the length-scale of the vortex is small compared with the radius of the planet, the force imbalance due to the meridional variation of the Coriolis force over the extent of the vortex is small and so the westward drift of the vortex is correspondingly small. Nonetheless, the drift speed is significant given the lifetime of many geophysical vortices.

The physical argument above can be made quantitative using centre-of-mass calculations based on the shallow-water equations governing the motion of a thin layer of fluid using the so-called β-plane approximation (see, for example, Nof 1981; Killworth 1983). This approximation, widely used in geophysical fluid dynamics since it is the simplest possible model for the effects of the sphericity of the Earth, involves writing the Coriolis parameter as a constant, f_0, based on some mean latitude, with a linear variation in the local north–south (y) coordinate, i.e. $f \approx f_0 + \beta y$. Here, β is a constant, which can be estimated from the dimensions of the planet: $\beta = 2\Omega \cos \phi_0 / a$, where a is the radius of the planet and ϕ_0 is the appropriate mean latitude. The β-plane approximation is valid only for small excursions of the vortex in the meridional (y) direction.

A similar argument to that above applied to cyclones in the Northern Hemisphere that have anticlockwise circulation leads to the erroneous conclusion that they drift eastward. It must be emphasized that the situation depicted in figure 4*a*, *b* is applicable only to vortices that consist of isolated lenses of fluid and, as such, are always

† Note there are other definitions of an isolated vortex. For instance, requiring that the net vorticity anomaly (i.e. circulation) be zero (see, for example, Llewellyn Smith 1997).

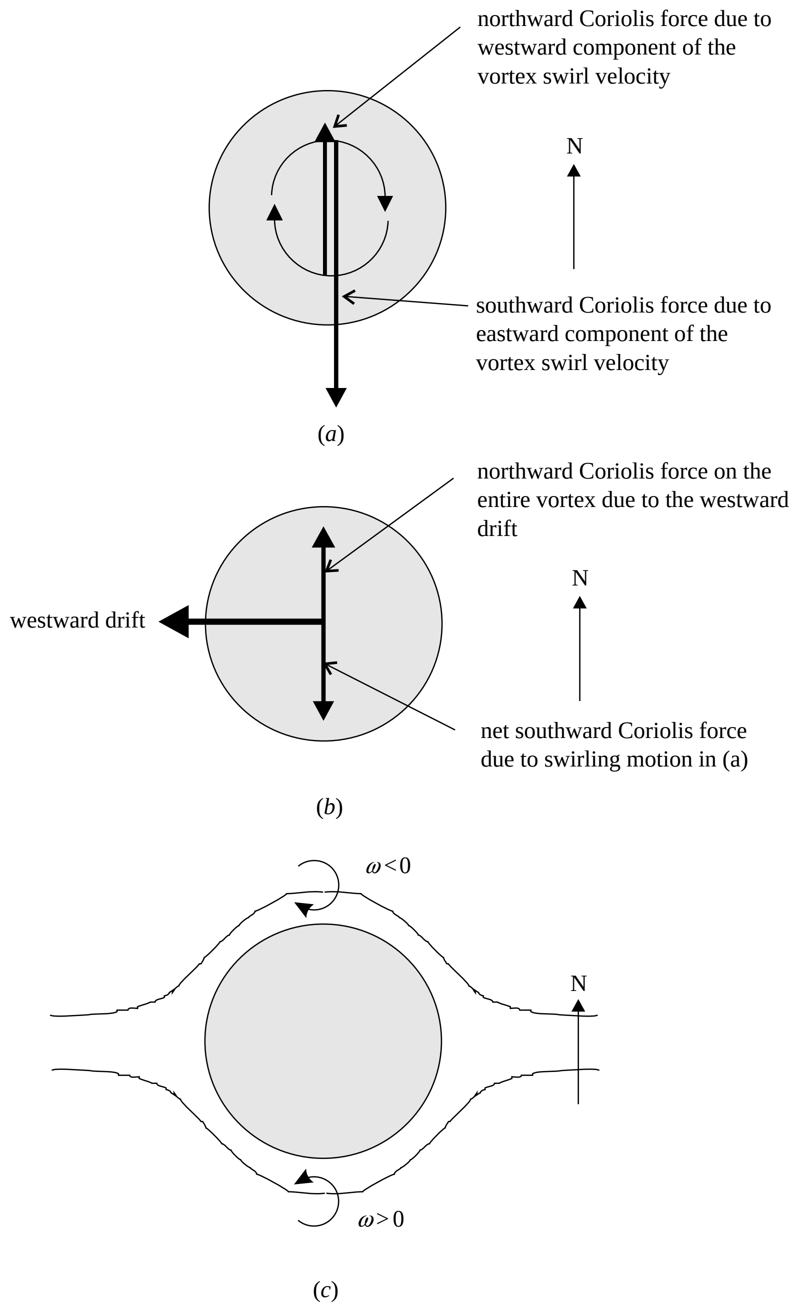

Figure 4. The westward drift mechanism for an isolated lens of fluid (an anticyclone) (*a*) and (*b*); and a cyclone (*c*) (see text for description).

necessarily anticyclones. The reason being that anticyclones correspond to anomalous high pressure and, for a single layer of fluid, this consists of a local thickening of the fluid, which decreases with distance from the central pressure, or fluid thickness, maximum. If this thickness decreases to zero, the vortex is an isolated 'blob' of fluid with finite volume. On the other hand, cyclones consist of an anomalous region of low pressure, or, equivalently, a local depression in a layer of fluid. Since the layer thickness increases with distance from the central pressure minimum, a cyclone cannot be isolated in the sense that it cannot be of finite volume and must, therefore, involve flow exterior to the vortex core. Figure 4*c* shows the situation for the case of a cyclone in the Northern Hemisphere. While the argument outlined above still applies to the core of the vortex (shaded), fluid external to this core must, as demanded by the conservation of potential vorticity (equation (2.2)), acquire relative vorticity as it is pushed around the vortex. Fluid forced northward increases f and so acquires negative ω, hence producing a local secondary clockwise circulation. Similarly, fluid forced south acquires positive ω, hence producing a local secondary anticlockwise circulation. The net effect of these secondary circulations surrounding the cyclone is to advect the cyclone west. Thus there are two competing effects: that shown in figure 4*b*, which tends to cause eastward drift, and the effect in figure 4*c* producing westward drift. Quantitative analysis shows that this latter effect always wins, and cyclones, like anticyclones, always drift west, albeit at a speed less than that of anticyclones. Note that westward drift for both cyclones and anticyclones also occurs in the Southern Hemisphere.

It is an illuminating exercise to account for the westward drift of a vortex on a rotating planet from the point of view of an inertial observer, i.e. one not fixed to the rotating planet. Such an observer cannot appeal to Coriolis forces, since such forces exist only in a rotating frame of reference. What, then, causes this westward drift? It has recently been shown (Nycander 1996) that the drift is analogous to the precession of a spinning top. Assuming steady motion, he obtained a formula for the precessional speed of the disc, which reduced to that obtained by, for example, Nof (1981) and Killworth (1983) using the shallow-water equations in a rotating frame of reference. McDonald (1998*b*), using Lagrangian rigid-body theory, extended Nycander's results to include time-dependent effects and showed that the disc may also undergo nutation: a periodic motion superimposed on the westward drift. These are sometimes known as inertial oscillations and are observed in the trajectories of geophysical vortices (e.g. in Meddy trajectories, see Armi *et al.* (1989)).

3. Evolution of a vortex on a β-plane

Section 2 indicates that a vortex in *steady* motion will propagate westward. How does a vortex starting from rest achieve such a steadily propagating state, if indeed it does? Much research has been aimed at understanding the evolution of isolated vortices on a rotating planet, or, more generally, in the presence of a gradient in the background potential vorticity. Analytical, numerical and experimental methods have been used to tackle this initial-value problem.

A common approach has been to use *quasi-geostrophic* theory together with the β-plane approximation. Quasigeostrophic theory represents an approximation to the shallow-water equations and is valid only for small Rossby number and when variations in layer thickness are small compared with the average thickness. This latter requirement precludes the study of the isolated blobs of fluid (discussed in § 2) since these necessarily have large variations in layer thickness. The use of quasi-geostrophic dynamics, while still nonlinear, yields a much simpler mathematical system than the

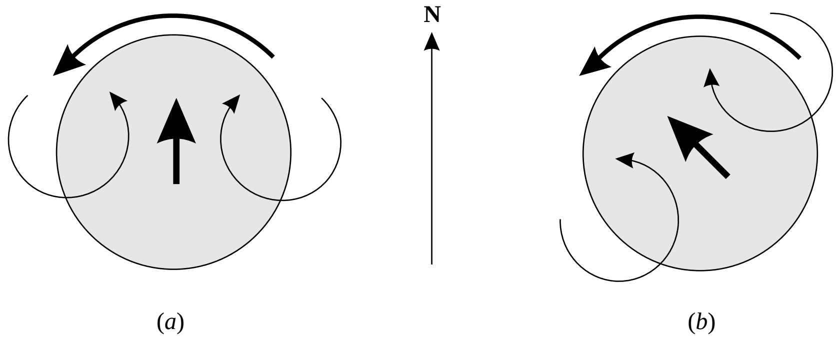

Figure 5. Formation of the β-gyres. The main part of the cyclonic vortex is shaded. In (*a*), the initial motion of the vortex is north and some time later, in (*b*), the motion of the vortex is northwest.

more general shallow-water equations. For example, in quasi-geostrophic theory it is possible to express the dynamics in one equation for one variable (the stream function), in contrast to the shallow-water equations, which consist of three nonlinear, coupled equations in three unknowns. Though the small Rossby number assumption is acceptable for many geophysical vortices, the assumption requiring small deviations in the layer thickness is, unfortunately, not so good, particularly for ocean vortices. Nonetheless, much can be learned about the behaviour of vortices using quasi-geostrophic theory, since it retains the essential ingredients of nonlinearity and the β-effect. An added bonus in such studies is that they are valid for arbitrary *Ro* in the special case when there is no deformation in the layer thickness; that is, the fluid is bounded above and below by rigid boundaries. Tropical cyclones are reasonably well modelled using such boundary conditions.

The introduction of the paper by Llewellyn Smith (1997) gives a good summary of previous studies on the initial-value problem of a circular vortex on a β-plane. What is clear from these studies is that the first stage of the evolution of the vortex is the development of a secondary dipole circulation, known as the β-gyres. A dipole is a pair of counter-rotating vortices that mutually advect each other, thus providing a mechanism for self-propagation. Here they develop as a result of potential vorticity conservation. For example, the anticlockwise circulation associated with a Northern Hemisphere cyclone induces a northward (southward) motion to its east (west) (see figure 5). Recalling (cf. figure 4*c*) that northward displacement of fluid results in negative relative vorticity production and southward displacement results in positive relative vorticity production, it follows that clockwise circulation develops to the east of the vortex and anticlockwise circulation develops to the west, i.e. a dipole is generated. The sense of the dipole is such that, initially, it advects the dipole northward (figure 5*a*). In turn, the circulation of the vortex rotates the axis of the dipole anticlockwise as shown in figure 5*b*. Thus, the cyclone moves northwestward along a curved trajectory. A similar argument shows that a Northern Hemisphere anticyclone moves southwestward. This behaviour has been demonstrated experimentally (see, for example, Carnevale *et al*. 1991) and numerically (see, for example, Sutyrin *et al*. 1994) and is reasonably well understood. For instance, the dependence of the trajectories on the initial strength and distribution of vorticity has been calculated analytically for several special cases (see, for example, Reznik 1992; Sutyrin & Flierl

1994; Reznik & Dewar 1994; Llewellyn Smith 1997). Many geophysical vortices are intense: that is, their vorticity anomaly has a much greater magnitude than the change in vorticity over the extent in the vortex due to β. The ratio of the two vorticities gives a small parameter that the aforementioned analytical studies exploit in developing a perturbation method. Thus, for intense vortices, the β-gyres are relatively weak vortices compared with the vortex itself and so the translation velocity of the vortex is small compared with a typical swirl velocity.

The next stage in the evolution is less well understood. This is because after times of the order of the time taken for the vortex to propagate a distance comparable with its width, the effects of Rossby wave radiation become important (i.e. *ca.* 2–3 days for an atmospheric vortex and *ca.* 10 days for an oceanic vortex). Recall from § 2 that the meridional gradient in the Coriolis parameter, as represented by the β-effect for instance, imparts an elasticity to the fluid enabling the propagation of Rossby waves. As the β-gyres cause the vortex to move, the resulting disturbance to the background potential vorticity field leads to the radiation of Rossby waves. This poses a difficult problem for theoreticians: the Rossby wave wake must drain energy from the vortex affecting its intensity and structure, and this, in turn, must influence the form of the Rossby wave wake. Perturbations induced by wave radiation, though weak, may have considerable effect over large times. Most attempts to study this stage of the evolution assume that the vortex is propagating quasi-steadily in a mainly westward direction. By quasi-steady it is meant that the vortex responds only slowly to the radiation, and, at any instant, the vortex and wake may be considered steady. A solution can then be found for an intense vortex with a small-amplitude Rossby wave wake. Calculating, for example, the energy flux in the wake and relating it to the rate of change of the vortex energy, enables the response of the vortex to be calculated. For example, Flierl (1984), Korotaev & Fedotov (1994) and McDonald (1998*a*) have adopted such an approach and demonstrated that an effect of the radiation is to cause a slow meridional drift in the vortex and, in the latter case, a slow shrinkage of the vortex core.

There are, however, some problems with this approach (as noted by Reznik & Grimshaw (2000)). At the time when wave radiation effects become important, the meridional and westward vortex velocities are comparable, and the vortex velocity is not predominantly westward as the quasi-steady theory requires. Moreover, the quasi-steady Rossby wave wake can be shown to have infinite energy, which is clearly unacceptable since the wake energy must originate from the vortex itself, which, of course, originally had finite energy.

Very recently (Reznik & Grimshaw 2000), there has been progress in the theoretical description of this stage of the vortex evolution without recourse to the quasi-steady approximation. Starting with an intense, divergent (i.e. the layer thickness is non-constant), quasi-geostrophic vortex modelled as a patch of constant potential vorticity, they calculate higher-order corrections to the dipolar β-gyres. In particular, they demonstrate that development and intensification of quadrupole and secondary axisymmetric components lead to the deceleration of the vortex. Taking into account energy and enstrophy (i.e. the global total of the square of the vorticity) conservation, they are able to show that the vortex decays such that its lifetime is proportional to its strength.

Recent progress has also been made in the high-resolution numerical modelling of quasi-geostrophic vortices on the β-plane. Lam & Dritschel (2000) use the recently

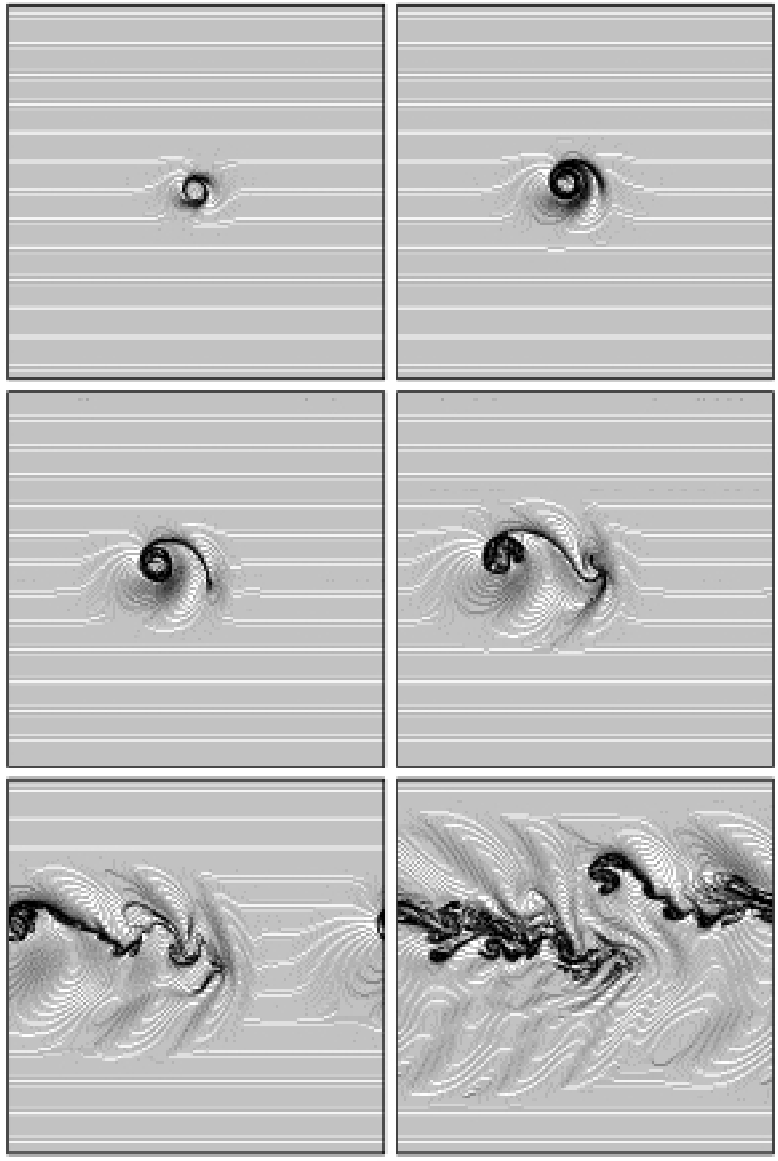

Figure 6. Evolution of a moderate-intensity, quasi-geostrophic vortex on the β-plane (after Lam & Dritschel 2000). Contours of potential vorticity taken at 1, 6, 11, 21, 41 and 81 vortex rotation times are shown. North is vertically upward and the domain is periodic in the east–west direction.

developed contour-advective semi-Lagrangian (CASL) algorithm to model the evolution of divergent, quasi-geostrophic vortex patches, thus complementing the analytical results of Reznik & Grimshaw (2000). The CASL algorithm is based on potential vorticity conservation and efficiently handles both the patch of constant vorticity representing the vortex (unlike spectral methods such as Sutyrin *et al*. (1994)) and the linear meridional variation in the background potential vorticity introduced by the β-effect. The numerical experiments of Lam & Dritschel (2000) demonstrate two principal processes in the vortex evolution:

(i) the redistribution of the ambient potential vorticity by the vortex; and

(ii) the deformation of the vortex boundary.

They show that the westward drift speed increases with increasing strength of the vortex and, like Reznik & Grimshaw (2000), highlight the importance of an axisymmetric perturbation of opposite relative vorticity to the vortex. The meridional motion is shown exclusively to be a result of the trailing wake of the vortex and is maximum for intermediate strength vortices. Figure 6 shows an example result of Lam & Dritschel (2000) for the case of an intermediate strength cyclonic vortex. The potential vorticity field is shown, which initially consists of the linear increase in the northward (y) direction and a circular patch of uniform vorticity. As expected, the vortex, being cyclonic, drifts northwest. The effects of the Rossby wave radiation are clear, as manifested by the undulating potential vorticity contours in the far field. Of particular importance is the shedding of vorticity in the 'turbulent wake' of the vortex, which has a significant effect on the vortex trajectory. Such an effect, while important, is not modelled in the analytical study of Reznik & Grimshaw (2000) and it is difficult to see how it could be (see § 4).

Figure 7 shows a laboratory experiment in which the wake shed by the moving vortex is evident. In the laboratory, the meridional variation in the ambient potential vorticity on a spherical planet is mimicked by a sloping bottom, such that the direction from deep to shallow water (the y-direction) is equivalent to the south to north direction (cf. § 2 and figure 3). Another feature of the experiment shown in figure 7 is the presence of a sudden change of depth midway along the tank, as indicated by the vertical dark line. That is, there is an escarpment running north–south separating shallow water on the right from deep water on the left. Initially, the vortex drifts to the west (i.e. decreasing x) until it reaches the escarpment, where it appears to be reflected. Such vortex–topography interaction merits further study.

4. Future developments

There is still much to understand about the behaviour of geophysical vortices even in the relatively simple problem of quasi-geostrophic vortex evolution on the β-plane. As the numerical experiments of Lam & Dritschel (2000) demonstrate, the shedding of vorticity in a 'vortex sheet' plays a significant role in their evolution. The fact that the meridional velocity is maximal for intermediate strength is interesting and indicates that, in this case, the vortex and the vorticity in the trailing vortex sheet pair up as a dipole to drive significant meridional motion. This is also consistent with the numerical results of McDonald & Dunn (1999) and the experimental results of Zavala Sanson *et al*. (1999), which show that when the strength of the vortex is of comparable strength to the background potential vorticity gradient, dipole formation appears to be a ubiquitous, dominant and robust feature in the overall dynamics. Is it possible to analytically model the interaction of this vortex sheet with the vortex? The difficulty here is that, in the interesting case when the vorticity in the wake is comparable to the vorticity of the vortex, their interaction is nonlinear, and, moreover, the wake energy remains localized near the vortex and is not radiated away

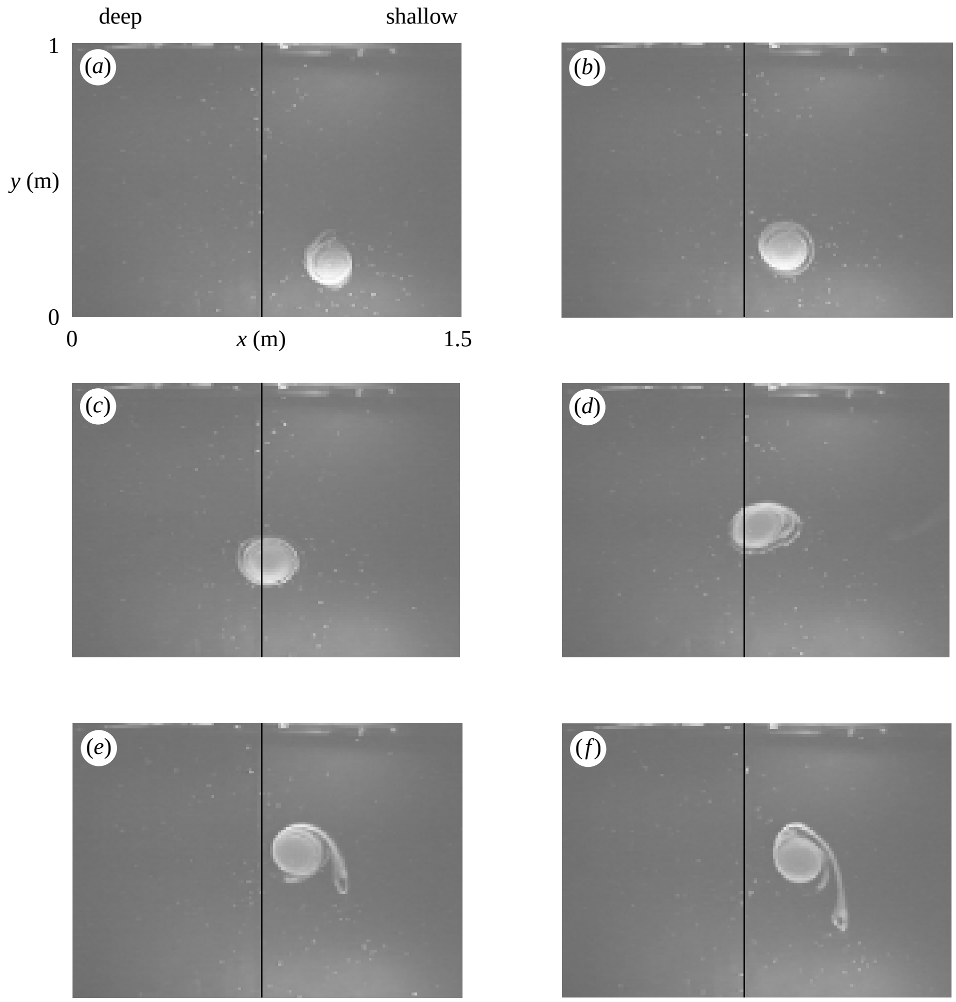

Figure 7. Laboratory experiment showing the initial 'westward' drift of a vortex (cf. figure 6). The dye concentration is shown. The fluid depth decreases uniformly toward increasing y, corresponding to the northward direction. Also there is an escarpment (indicated by the dark vertical line) separating deep and shallow regions. The vortex appears to be reflected when it reaches the escarpment at $t = 80$ s.

to infinity as it is for Rossby waves. Of course, a successful model would be desirable in order to gain quantitative predictions for the vortex trajectory, in particular its meridional displacement.

Perhaps the greatest immediate challenge to the modelling community comes from the need to relax the restrictions imposed by the quasi-geostrophic assumption. Ocean eddies in particular have variations in layer thickness sufficiently large to invalidate quasi-geostrophic theory. There is then a need to use more appropriate dynamical theories using the shallow-water equations for example. From a numerical modelling point of view, there has been recent progress in developing the CASL algorithm for the shallow-water equations (see Dritschel *et al*. 1999).

This offers the exciting possibility of the high-resolution study of shallow-water (as opposed to quasi-geostrophic) vortex evolution on the β-plane. An important property of the shallow-water equations is that there is an asymmetry—not present in the quasi-geostrophic equations—in the behaviour of cyclones and anticyclones (cf. § 2 and figure 4 regarding the different drift speeds of cyclones and anticyclones). This asymmetry may explain the observed bias of the Earth's oceans and the atmosphere of Jupiter toward anticyclones over cyclones (see, for example, McWilliams 1985). Indeed, McDonald (1998*a*) used the shallow-water equations to demonstrate that cyclones will radiate Rossby waves on the β-plane, whereas anticyclones will not. Invoking the quasi-steady hypothesis, McDonald obtained an explicit time-scale for the decay of cyclones and showed that the cyclone's radius decreased in response to the radiation. However, the quasi-steady approximation employed by McDonald (1998*a*) is questionable (see discussion in § 3), and an analysis similar to Reznik & Grimshaw (2000) may lead to more believable results. A possible analytical approach may be to consider the shallow-water equations *near* (as measured by some small parameter) the quasi-geostrophic limit, so as to retain symmetry breaking between the cyclone and anticyclone behaviour. The smallness of this parameter could then be exploited to find the correction to the quasi-geostrophic behaviour.

Most of this paper has been concerned with the behaviour of vortices comprising a single layer of fluid. However, many vortices, again particularly those in the ocean, are best modelled by two or more layers of fluid. For example, Agulhas eddies have well-defined surface signatures and it may be expected that they are surface-trapped features and are adequately modelled as single-layer 'blobs'. Observations show that some Agulhas eddies are influenced by the Walvis Ridge (a steep sea-floor topographic feature); some are able to propagate over the ridge but some are trapped by the ridge (Byrne *et al*. 1995). This suggests that there is a deep lower layer flow associated with Agulhas eddies and that it is appropriate for them to be modelled by two (at least) layers, each of which may contain vorticity anomalies. Models are currently being developed to study the interaction of two-layer vortices with topography, starting with relatively simple quasi-geostrophic theory, enabling, in the first instance, analytical work and simple numerical models to be used. This will lead to more dynamically complete multilayer shallow-water simulations. In the latter case, the CASL algorithm can be adapted to cope with multiple layers and the presence of topography. The aim of such studies being to determine, quantitatively, the effects of such factors as topographic shape and size on vortex trajectories and lifetimes.

Even the problem of the interaction of single-layer, *non*-quasi-geostrophic vortices with topography raises interesting questions. For instance, large-*Ro* vortices (i.e. rotation unimportant), such as those that occur in the surf zone, are known to propagate along contours of constant depth in opposite directions according to the sign of their vorticity. On the other hand, small-*Ro* vortices also propagate along contours of constant depth but in the *same* 'westward' direction, irrespective of the sign of their vorticity. It would be an interesting study to examine the effect of varying *Ro* from very small to very large values to see its effect on, for example, the direction and speed of propagation. Is there, for example, a critical Rossby number and form of topography such that the vortex is stationary?

Another aspect of vortex motion that has received little attention is the literature of multiple vortex interaction. This is relevant to Agulhas eddies and Gulf Stream rings, where vortices do not occur in isolation but rather in groups. In fact, Agulhas eddies form a train that propagates northwestward into the Atlantic. How does such an arrangement of vortices affect their propagation speed and rate of decay? For example, rather like designing the hull of a yacht to minimize its wave resistance, are Agulhas eddies arranged in such a manner as to minimize their Rossby wave drag? How do their collective wakes interact? Both analytical and numerical studies can be used to shed light on such questions. A good starting point in an analytical study would be to assume large-amplitude (intense) vortices separated at distances large compared with their own radii.

The above studies invariably use the β-plane approximation, which, recall, is only locally valid and its applicability is doubtful when applied to, for instance, Meddies, which are known to travel large meridional distances. Moreover, the consistency of the β-plane approximation has been questioned (Ripa 1997). Ideally, it is desirable to formulate vortex models in spherical geometry. Perhaps an approach similar to that of the rotating disc on a sphere (§ 2) could be used with the important inclusion of the effects of the ambient fluid (e.g. allowing for Rossby wave radiation). This may be most easily accomplished using a Lagrangian formulation of the shallow-water equations (see, for example, Salmon 1988) in spherical geometry analogous to McDonald's (1998*b*) rigid-body theory for the motion of a rigid disc on a sphere.

The scattering, rather than generation, of waves, especially Rossby waves, by vortices is a topic that has received little attention. Such waves are ubiquitous in geophysical fluids and, inevitably, vortices will encounter them. Can the life of a vortex be prolonged by the absorption of energy from the scattering of Rossby waves? Perhaps this explains the longevity of Jupiter's Great Red Spot. Such a study could, in the first instance, be done analytically using quasi-geostrophic theory.

Big advances in satellite observation technology and data-processing techniques leading, in particular, to their ability to resolve mesoscale ocean features such as vortices, are keenly anticipated within the next few years. Coupled with improving numerical techniques and a greater understanding of simple models through analytical work, the near future promises to be an exciting time in the important study of the motion of geophysical vortices.

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